DUNNINGTON CE PRIMARY SCHOOL

WRITTEN CALCULATIONS POLICY

Date of Policy:	November 2023
Date of Review:	November 2026
Member(s) of staff responsible:	Katie Priest and Rachel Dickenson
Governor(s) responsible:	School Improvement Committee
Signature of Chair of Governors:	
Date:	07.11.23

Calculations Policy

1. Rationale

Dunnington C.E. Primary School is committed to the engaging and progressive delivery of mathematics across the age ranges and throughout the curriculum. This policy blends current practices with the expectations of the Primary National Curriculum, which itself provides a structured and systematic approach to teaching number and calculation.

Although the main focus of this policy is on pencil and paper procedures for the four operations of addition, subtraction, multiplication and division, it is important to recognise that the ability to calculate mentally lies at the heart of numeracy. Mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it. In every written method there is an element of mental processing. Written recording both helps children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

Progressive calculation methods will be taught in each year group and the ultimate decision to move a child onto a new method of calculation lies with the teacher and rests on the student feeling confident and secure with the method they currently rely upon. The long-term aim is for children to be able to select an efficient method of their choice that is appropriate for a given task by asking themselves:

'Can I do this in my head?'
'Can I do this in my head using drawings or jottings?'
'Do I need to use a pencil and paper procedure?'
'Do I need a calculator?'

2. Aims

- To present the preferred calculation methods so that there is a consistent programme of teaching throughout the school with progression clearly shown.
- To ensure a steady progression of understanding in maths for children as they move through the school.
- To make clear the calculations involved in the four number operations for parents and pupils.
- To help communicate methods and solutions.
- To help the development of fluency in daily mental maths practice.
- To equip children with a 'toolkit' of mental, written and calculator methods that they understand and can use and apply correctly when faced with a problem or an unfamiliar mathematical context.
- Look for patterns and make connections between concrete, pictorial and abstract representations.
- To develop secure methods of calculation for children to explore their reasoning and problem-solving skills.

3. Whole School Approach to Calculation

To establish continuity and progression throughout the school, we have developed a consistent approach to the teaching of **written calculation** methods.

Mental methods will be clearly established. These will be based on a sound understanding of number and number facts and will include the following:

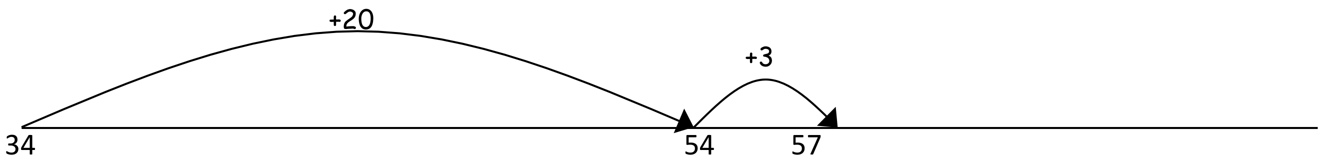
i. Instant recall of key number facts
e.g. pairs of numbers which make 10, doubles and halves to 20

ii. Using known facts to calculate unknown facts
e.g. 6+6 = 12 therefore 6+7 = 13, 24+10 = 34 therefore 24+9 = 33

iii. Understanding and using relationships between addition and subtraction to find answers and check results
e.g. 14+6 = 20 therefore 20-6 = 14

iv. Having a repertoire of mental strategies to solve calculations
e.g. doubles/near doubles, bridging 10 / bridging 20, adding 9 by +10 & -1

v. Making use of informal jottings and diagrams to assist in calculations with larger numbers
e.g. 34 + 23 = 57



vi. Solving one-step word problems (either mentally or with jottings) by identifying which number operation to use, drawing on their knowledge of number bonds and explaining their reasoning.

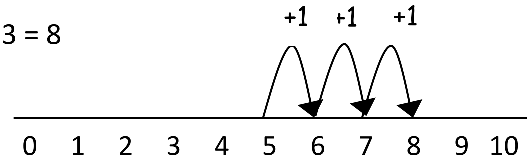
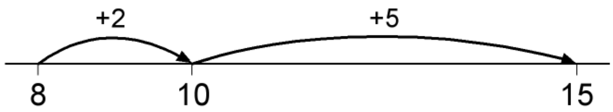
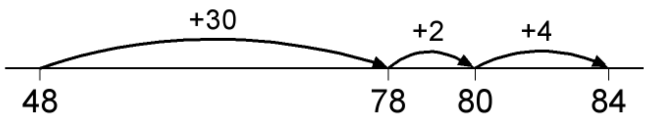
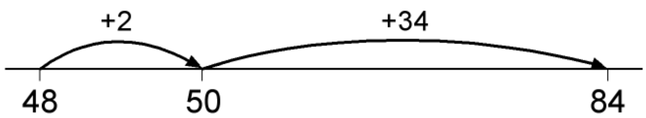
vii. Begin to record calculations in a horizontal format and explain mental calculations steps using numbers, symbols and/or words.

viii. Learn to estimate/ approximate to solve calculations
e.g. 29 + 30 can be rounded up to the nearest 10 i.e. 30 + 30, therefore the answer will be to near to 60

Place value will be taught mentally, beginning in Foundation Stage where number tracks are introduced and progressing to number lines in Years 1 and 2. The empty number line will also then be introduced to aid calculations.

Calculations Policy

4. Stages in Addition

Different Stages	**Examples in practice**
Stage 1: Combining sets • Combining 2 sets of objects into 1 group and counting practically.	For example for 6+2= the children may get 6 cubes, then 2 more and count how many altogether.
Stage 2: Informal jottings • Using pictures/dots – informal jottings. Then counting how many altogether.	5 + 3 = 8 * * * * * + * * *
Stage 3: Counting on • Counting on, on a number line with numbers on it.	5 + 3 = 8 
Stage 4: The empty number line • The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and units separately, often starting with the tens as this is the larger part of the number. • Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.	8 + 7 = 15  48 + 36 = 84 30 6  **OR:** 

Calculations Policy

<table>
<tr>
<td>
Stage 5: Partitioning

• The next stage is to record mental methods using partitioning. Add the tens and then the units to form partial sums and then add these partial sums.
• Partitioning both numbers into tens and units mirrors the column method where ones are places under ones and tens under tens. This also links to mental methods.
</td>
<td>
Record steps in addition using partitioning:

43 + 24=
40 + 20 = 60
3 + 4 =7
60 + 7 = 67

which is then recorded in a shorter form below
43 + 24 = 60 + 7 = 67

Partitioned numbers are then written under one another:
T U   T   U
43   =   40 +   3
+ 24   20 +   4
           60 +   7 = 67
</td>
</tr>
<tr>
<td>
Stage 6: Expanded methods in columns

• Move on to a layout showing the addition of the tens to the tens and the units to the units separately. At this stage, ask the children to start by adding the units first always.

• This is very important as it will need to be done in this order when moving onto the compact method.

• The addition of the tens in the calculation 47 + 76 is described in the words ‘forty plus seventy equals one hundred and ten’, stressing the link to the related fact ‘four plus seven equals eleven’.

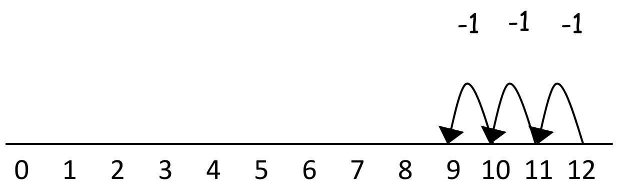
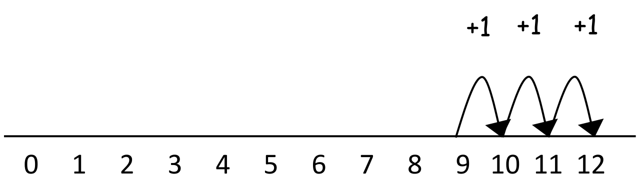
• The expanded column method leads children to the more compact method of addition so that they understand its structure and efficiency.
</td>
<td>
Write the numbers in columns.

Add the ones first:
T U
43
+ 24
7
60
67
</td>
</tr>
</table>

Calculations Policy

Stage 7: Compact column method	
• In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'. • Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits. • Column addition remains efficient when used with *larger whole numbers* and *decimals*.	T U 43 + 24 67 H T U 687 +248 935 1 1

5. Stages in Subtraction

Different Stages	**Examples in practice**
Stage 1: Removing objects from sets • Practically get a group of objects and take some away.	
Stage 2: Informal jottings • Draw a set of objects and then cross some out.	11 – 4 = 7 X X X X * * * * * * *
Stage 3: Counting back or counting on • Count back on a number line with numbers on, starting from the initial value in the sum. • Count on using a number line with numbers on, starting from the subtraction value to the initial value in the sum. • Both methods can be used when finding the difference between two given numbers.	12 – 3 = 9 by counting back:  12 – 9 = 3 by counting on: 

Calculations Policy

<table>
<tr>
<td>

Stage 4: The empty number line- Counting back

- The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 - 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.

- The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.

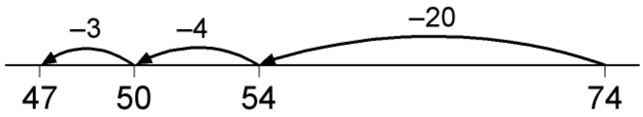
- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 - 12, 86 - 77 or 43 - 28.

</td>
<td>

Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

15 - 7 = 8

74 - 27 = 47 worked by **counting back**:



</td>
</tr>
<tr>
<td>

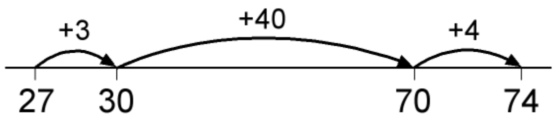
Stage 5: The empty number line- Counting on

- The mental method of counting up from the smaller to the larger number can be recorded using number lines. Children usually find it easiest to make the first jump to the next 10.

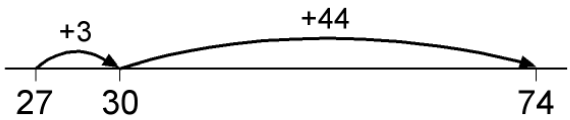
- The number of jumps will vary. For some children, they will find it comfortable to make only two jumps along the line. Others will need more. Children usually find it easiest to make the first jump to the next 10.

</td>
<td>

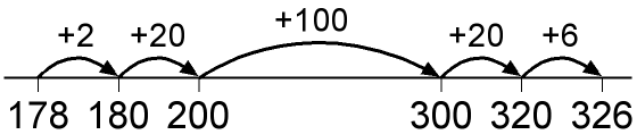
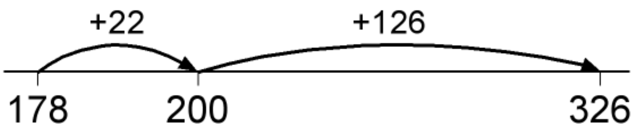
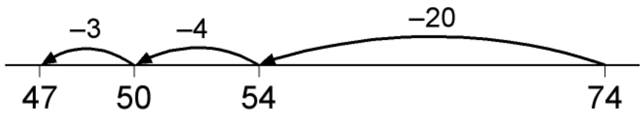
74 - 27 = 47 worked by **counting up**:



As children become more confident, they will often require less jumps:

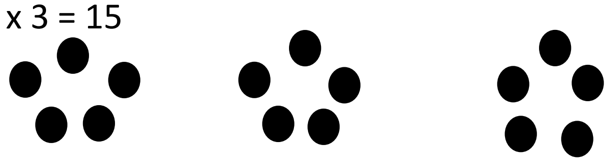
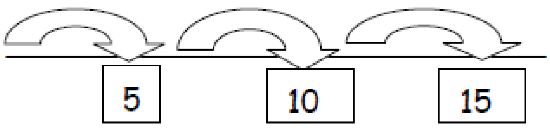
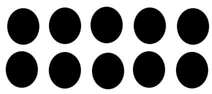


</td>
</tr>
</table>

	For 3 digit numbers: 326 – 178=148  Leading to: 
Stage 6: Partitioning • Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 -27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.	Subtraction can be recorded using partitioning: 74 - 27 = 74 – 20 = 54 54 – 7 = 47 This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line. 
Stage 7: Expanded layout, leading to column method • Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens. • This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.	74 – 27= $\begin{array}{rr} & \text{T}\qquad\text{U}\\ & 70+4\\ - & 20+7\\ \hline \end{array}$ $\quad$ $\begin{array}{rr} & \overset{60}{\cancel{70}}+\overset{14}{\cancel{4}}\\ - & 20+7\\ \hline & 40+7\end{array}$ $\quad\Rightarrow\quad$ $\begin{array}{rr} & \overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}}\\ - & 2\ \ 7\\ \hline & 4\ \ 7\end{array}$ **For 3 digits with 'borrowing' from the hundreds to the tens and the tens to the ones:** 741- 367 = $\begin{array}{rr} & \text{H}\quad\text{T}\quad\text{U}\\ & 700+40+1\\ - & 300+60+7\\ \hline \end{array}$ $\quad$ $\begin{array}{rr} & \text{H}\qquad\text{T}\qquad\text{U}\\ & \overset{600}{\cancel{700}}+\overset{130}{\cancel{40}}+\overset{11}{\cancel{1}}\\ - & 300+60+7\\ \hline & 300+70+4\end{array}$ $\quad\Rightarrow\quad$ $\begin{array}{rr} & \overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ \overset{11}{\cancel{1}}\\ - & 3\ \ 6\ \ 7\\ \hline & 3\ \ 7\ \ 4\end{array}$

• The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.	**For 4 digits including dealing with zeros when 'borrowing':** 5008 – 1257 = Th H T U 4000 900 100 ~~5000 + 000~~ + ~~00 + 8~~ - 1000 + 200 + 50 + 7 3000 + 700 + 50 + 1 Th H T U 9 4 ~~10~~ 10 5 0 0 8 - 1 2 5 7 3 7 5 1

6. Stages in Multiplication

<u>Different Stages</u>	<u>Examples in practice</u>
Stage 1: Grouping • Grouping objects in repeated groups/patterns and counting	5 x 3 = 15 
Stage 2: Repeated addition • 5 times 3 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3 Repeated addition can be shown easily on a number line.	5 x 3 = 5 + 5 + 5 = 15 5 5 5 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 
Stage 3: Arrays	3 x 5 =15 or 5 x 3 = 15 

<table>
<tr>
<td>Stage 4: Mental multiplication using partitioning
Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.</td>
<td>Informal recording might look like:

43 x 6 =
43
40 + 3
↓ ↓ × 6
240 + 18 = 258

More formal recording mental multiplication using partitioning:

15 x 5
= 10 x 5 = 50
= 5 x 5 = 25
= 50 + 25 =75</td>
</tr>
<tr>
<td>Stage 5: The grid method
As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.
It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.
The grid method may be the main method used by children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.</td>
<td>38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266
<table><tr><td>×</td><td>7</td></tr><tr><td>30</td><td>210</td></tr><tr><td>8</td><td>56</td></tr><tr><td></td><td>266</td></tr></table></td>
</tr>
<tr>
<td>Stage 6:Expanded column multiplication
The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
Children should describe what they do by referring to the actual values of the digits in the columns as outlined in the brackets.</td>
<td>H T U
1 4 7
x 4
2 8 (4 x 7)
1 6 0 (4 x 40)
4 0 0 (4 x 100)
5 8 8</td>
</tr>
</table>

Stage 7a: Column multiplication

- The recording is reduced further, with carry digits recorded below the line.
- If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 6.

$$
\begin{array}{cccc}
 & H & T & U \\
 & 1 & 4 & 7 \\
\times & & & 4 \\
\hline
 & \mathbf{5} & \mathbf{8} & \mathbf{8} \\
\hline
 & _1 & _2 &
\end{array}
$$

Stage 8i: Two-digit by two-digit products

- Extend to TU × TU, asking children to estimate first.
- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.

56 × 27 is approximately 60 × 30 = 1800.

×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

Stage 8ii:

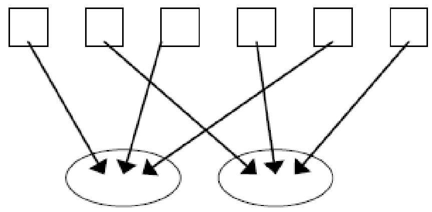
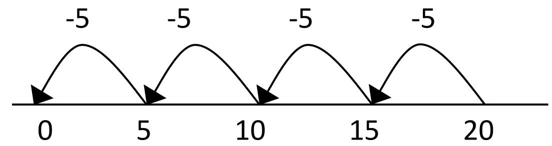
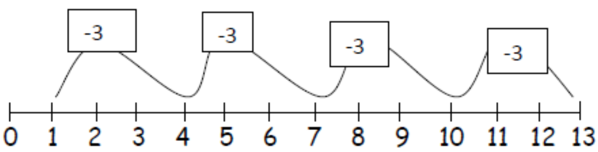
- Reduce the recording, showing the links to the grid method above.

56 × 27 is approximately 60 × 30 = 1800.

$$
\begin{array}{rrrr}
 & \text{T U} & & \\
 & 56 & & \\
\times & \underline{27} & & \\
 & 1000 & & 50 \times 20 = 1000 \\
 & 120 & & 6 \times 20 = 120 \\
 & 350 & & 50 \times 7 = 350 \\
 & \underline{42} & & 6 \times 7 = 42 \\
 & \underline{1512} & & \\
 & 1 & &
\end{array}
$$

Stage 8iii: • Reduce the recording further. • The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally. • Children who are secure with multiplication for TU x U and TU x TU should have little difficulty in using the same method for HTU x TU.	56 × 27 is approximately 60 × 30 = 1800. $\begin{array}{rrl} & \text{T U} & \\ & 56 & \\ \times & \underline{27} & \\ & 1120 & 56 \times 20 \\ & \underline{392} & 56 \times \;\, 7 \\ & \underline{1512} & \\ & 1\;\; & \end{array}$

7. Stages in Division

<u>**Different Stages**</u>	<u>**Examples in practice**</u>
Stage 1: Using jottings • Sharing equally or grouping objects	6 sweets shared between 2 people, how many do they each get? **Sharing equally:**  **Grouping:** 
Stage 2: Repeated subtraction • Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar. • Children should also move onto calculations involving remainders through repeated subtraction.	**Repeated subtraction using a number line:** 20 ÷ 5 = 4  13 ÷ 3 = 4 r 1 

Stage 3: Mental division using partitioning • Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient. • Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention. • Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by6.	One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining ones, then divide each part separately. Informal recording for 84 ÷ 7: 84 70 + 14 $\downarrow$ $\downarrow$ ÷ 7 10 + 2 = 12 In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.
Stage 4: 'Expanded method' for TU ÷ U and HTU ÷ U • This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. • For TU ÷ U there is a link to the mental method. • As you record the division, ask: 'How many nines in 90?' or 'What is 90 divided by 9?' • Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same.	97 ÷ 9 9)97 − 90 9 × 10 7 Answer: 10 R 7

- Chunking is useful for reminding children of the link between division and repeated subtraction.
- However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.
- The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that 'trap' the HTU dividend.
- Estimating has two purposes when doing a division: to help to choose a starting point for the division AND to check the answer after the calculation.
- Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.

$$
\begin{array}{rrl}
 & 6\overline{)196} & \\
- & \underline{60} & 6 \times 10 \\
 & 136 & \\
- & \underline{60} & 6 \times 10 \\
 & 76 & \\
- & \underline{60} & 6 \times 10 \\
 & 16 & \\
- & \underline{12} & 6 \times \underline{2} \\
 & 4 & 32 \\
\text{Answer:} & & 32\text{ R }4
\end{array}
$$

To find 196 ÷ 6, we start by multiplying 6 by 10, 20,30, ... to find that 6 × 30 = 180 and6 × 40 = 240.

The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40.

Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4.

$$
\begin{array}{rrl}
 & 6\overline{)196} & \\
- & \underline{180} & 6 \times 30 \\
 & 16 & \\
- & \underline{12} & 6 \times \underline{2} \\
 & 4 & 32 \\
\text{Answer:} & & 32\text{ R }4
\end{array}
$$

The quotient 32 (with a remainder of 4) lies between 30 and 40 as predicted.

Stage 6: Short division of HTU ÷ U

- Short' division of HTU ÷ U can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction.

- The accompanying pattern is 'How many threes in 290?' (the answer must be a multiple of 10). This gives 90 threes or 270, with 20 remaining. We now ask: 'How many threes in 21?' which has the answer 7.

- Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.

- For most children this will be at the end of Year 5 or the beginning of Year 6.

The short division is recorded as:

$$\begin{array}{r} 9\;7 \\ 3\overline{)2\;9\;{}^{2}1} \end{array}$$

And can be explained like this:

$$3\overline{)290+1} = \begin{array}{r} 90+7 \\ 3\overline{)270+21} \end{array}$$

The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that a total of 21 ones are to be divided by 3.

The 97 written above the line represents the answer: 90 + 7,or 9 tens and 7 ones.

Children are often encouraged to record their multiplication facts of the dividing number to aid in their calculation i.e. 3, 6, 9, 12, 15, 18, 21, 24, ***27****, 30 therefore when calculating 29÷3, it is clear that 3 divides into 29 a total of 9 times with a remainder of 2.*

Stage 7: Long division

- The next step is to tackle HTU ÷ TU, which for most children will be in Year 6.
- The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient. Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.

How many packs of 24 can we make from 560 biscuits?
Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

$$
\begin{array}{rrl}
 & 24\overline{)\,560} & \\
20 & -\underline{480} & 24\times 20 \\
 & 80 & \\
3 & \underline{72} & 24\times 3 \\
 & 8 &
\end{array}
$$

Answer: 23 R 8

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

$$
\begin{array}{r}
23 \\
24\overline{)\,560} \\
-\underline{480} \\
80 \\
-\underline{72} \\
8
\end{array}
$$

Answer: 23 R 8